

Hong Kong Mathematics Olympiad (2007 – 2008)

Heat Event (Individual)

香港數學競賽 (2007 – 2008)

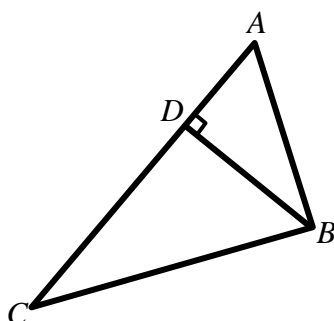
初賽項目(個人)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 如圖一， ABC 為一個三角形且 $AB = 13\text{cm}$ 、 $BC = 14\text{cm}$ 及 $AC = 15\text{cm}$ 。 D 為 AC 上的一點使得 $BD \perp AC$ 。若 CD 比 AD 長 $X\text{cm}$ ，求 X 的值。

In Figure 1, ABC is a triangle, $AB = 13\text{cm}$, $BC = 14\text{cm}$ and $AC = 15\text{cm}$. D is a point on AC such that $BD \perp AC$. If CD is longer than AD by $X\text{cm}$, find the value of X .



圖一

Figure 1

2. 已知梯形 $PQRS$ 的邊長分別為 $PQ = 6\text{cm}$ 、 $QR = 15\text{cm}$ 、 $RS = 8\text{cm}$ 及 $SP = 25\text{cm}$ ，並有 $QR \parallel PS$ 。若 $PQRS$ 的面積為 $Y\text{cm}^2$ ，求 Y 的值。

Given that a trapezium $PQRS$ with dimensions $PQ = 6\text{cm}$, $QR = 15\text{cm}$, $RS = 8\text{cm}$ and $SP = 25\text{cm}$, also $QR \parallel PS$. If the area of $PQRS$ is $Y\text{cm}^2$, find the value of Y .

3. 已知 x_0 及 y_0 為正整數且滿足方程 $\frac{1}{x} + \frac{1}{y} = \frac{1}{15}$ 。若 $35 < y_0 < 50$ 及 $x_0 + y_0 = z_0$ ，求 z_0 的值。

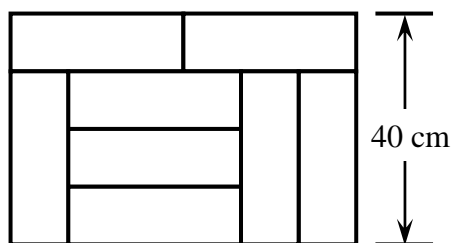
Given that x_0 and y_0 are positive integers satisfying the equation $\frac{1}{x} + \frac{1}{y} = \frac{1}{15}$. If $35 < y_0 < 50$ and $x_0 + y_0 = z_0$, find the value of z_0 .

4. 設 a 、 b 、 c 和 d 為實數。若 $\frac{a}{b} = \frac{1}{2}$ ， $\frac{b}{c} = \frac{3}{2}$ ， $\frac{c}{d} = \frac{4}{5}$ 及 $\frac{ac}{b^2 + d^2} = e$ ，求 e 的值。

Let a, b, c and d be real numbers. If $\frac{a}{b} = \frac{1}{2}$, $\frac{b}{c} = \frac{3}{2}$, $\frac{c}{d} = \frac{4}{5}$ and $\frac{ac}{b^2 + d^2} = e$, find the value of e .

5. 如圖二，利用 8 個相同的小長方形能拼出一個大的長方形。已知在大長方形中較短的邊長為 40 cm。若小長方形的面積是 $A \text{ cm}^2$ ，求 A 的值。

In Figure 2, the large rectangle is formed by eight identical small rectangles. Given that the length of the shorter side of the large rectangle is 40 cm and the area of the small rectangle is $A \text{ cm}^2$, find the value of A .

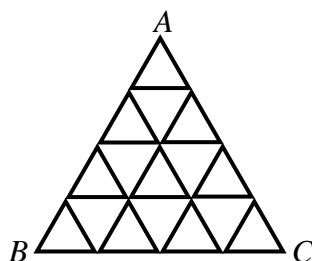


圖二

Figure 2

6. 在圖三中， $\triangle ABC$ 為等邊三角形。它由多個相同的等邊三角形組成。若圖中共有 N 個等邊三角形，求 N 的值。

In Figure 3, $\triangle ABC$ is an equilateral triangle. It is formed by several identical equilateral triangles. If there are altogether N equilateral triangles in the figure, find the value of N .



圖三

Figure 3

7. 設 r 為方程 $\frac{4}{y+1} + \frac{5}{y-5} = -\frac{3}{2}$ 的較大實根。求 r 的值。

Let r be the larger real root of the equation $\frac{4}{y+1} + \frac{5}{y-5} = -\frac{3}{2}$. Find the value of r .

8. 設 x 為有理數及 $w = \left| x + \frac{2007}{2008} \right| + \left| x - \frac{2007}{2008} \right|$ 。求 w 的最小可能值。

Let x be a rational number and $w = \left| x + \frac{2007}{2008} \right| + \left| x - \frac{2007}{2008} \right|$. Find the smallest possible value of w .

9. 設 m 和 n 為正整數。已知表達式 $\left(\left(\left((2)^2 \right)^{\cdot^{\cdot^{\cdot}}} \right)^2 \right)^{\cdot^{\cdot^{\cdot}}} = \left(\left(\left((4)^4 \right)^{\cdot^{\cdot^{\cdot}}} \right)^4 \right)^{\cdot^{\cdot^{\cdot}}}$ 含有 m 個 2 及 n 個 4。若 $k = \frac{m}{n}$ ，求 k 的值。

Let m and n be positive integers. Given that the number 2 appears m times and the number 4 appears

n times in the expression $\left(\left(\left((2)^2 \right)^{\cdot^{\cdot^{\cdot}}} \right)^2 \right)^{\cdot^{\cdot^{\cdot}}} = \left(\left(\left((4)^4 \right)^{\cdot^{\cdot^{\cdot}}} \right)^4 \right)^{\cdot^{\cdot^{\cdot}}}$. If $k = \frac{m}{n}$, find the value of k .

10. 求 $\log_2(\sin^2 45^\circ) + \log_2(\cos^2 60^\circ) + \log_2(\tan^2 45^\circ)$ 的值。

Find the value of $\log_2(\sin^2 45^\circ) + \log_2(\cos^2 60^\circ) + \log_2(\tan^2 45^\circ)$.